Incomplete collisions in strongly dispersion-managed return-to-zero communication systems

Mark J. Ablowitz, Andrew Docherty, and Toshihiko Hirooka

Department of Applied Mathematics, University of Colorado at Boulder, Boulder, Colorado 80309-0526

Received January 13, 2003

Incomplete collisions in wavelength-division-multiplexed return-to-zero transmission systems are analyzed by asymptotic methods. Formulas for frequency and timing shifts are obtained. The results agree with direct numerical calculations. © 2003 Optical Society of America

OCIS codes: 060.5530, 060.2330.

In wavelength-division-multiplexed transmission systems, collisions between pulses are inevitable. These collisions change the average frequency and time position of a pulse. So-called complete collisions have been studied extensively in both classic[1] and dispersion-managed (DM) systems.[2,3] Incomplete collisions (ICs) occur in realistic systems when the pulses initially overlap or are sufficiently close to one another to interact. ICs can give rise to large residual frequency shifts (FSs) and timing shifts (TSs) in both classic[4] and DM soliton systems.[5,6]

We describe, for the first time to our knowledge, an analytical method based on asymptotic analysis of the cross-phase integrals that represent the FSs and TSs with which to calculate ICs for strongly DM return-to-zero communication systems. The method presented is widely applicable and can be used to study both incomplete and complete collisions in quasi-linear (QL) and DM soliton systems. A wide range of experimental systems has been shown to operate in the QL regime.[7] For illustration, we concentrate on the more-complicated QL system in which all collisions are incomplete. For the typical QL system that we consider here, the length of the collision process (the distance over which mini collisions occur) is 32 Mm, which is significantly longer than a typical transoceanic system length of 10 Mm. It turns out that for QL systems ICs are not so damaging as they are in classic soliton systems. For QL systems we are able to derive elementary formulas that lead to realistic upper bounds for ICs in QL systems. These formulas can be used to rapidly and efficiently compute upper bounds for FSs and TSs in wavelength-division multiplexing systems.

To begin the analysis of collisions of QL pulses in a strongly DM system we transform the standard nonlinear Schrödinger equation into a dimensionless equation by using the equations $t = t_{\mathrm{ret}}/t_*$, $z = z_{\mathrm{lab}}/z_*$, $u = E/\sqrt{g(z)P_*}$, and $D = -k''/k_*''$, with normalization parameters denoted by a subscript $*$, where t_{ret} and z_{lab} are the retarded time and the propagation distance, respectively, and E denotes the slowly varying envelope of the optical field. Typical values used here are $P_* = 1$ mW, $z_* = z_{\mathrm{NL}} = 1/(\nu P*) = 400$ km, $t_* = 12$ ps, $f_* = 83$ GHz, and $k_*'' = {t_*}^2/z_* = 0.36\ \mathrm{ps}^2/\mathrm{km}$, where ν is the nonlinear coefficient. We find that

$$iu_z + (1/2)D(z)u_{tt} + g(z)\,|u|^2 u = 0\,, \tag{1}$$

where $g(z)$ describes the power variation that is due to loss and lumped amplification and $D(z)$ is the local group-velocity dispersion. Both are periodic, with period $z_a = l_a/z_*$, where l_a is the amplifier spacing, which is assumed to be small: $z_a \ll 1$. Strong dispersion management is modeled by $D(z) = \langle D\rangle + \Delta(z/z_a)/z_a$, where $\langle D\rangle$ is the path-average dispersion and $\Delta(\zeta) = \{\Delta_1{:}0 \le |\zeta| \le \theta/2, \Delta_2{:}\theta/2 \le |\zeta| \le 1/2\}$ is the dispersion variation with zero average; here θ is the fraction of the map with dispersion Δ_1. Taking lumped erbium-doped fiber amplification, we have $g(z) = g_0 \exp(-2\Gamma z)$ and $g_0 = 2\Gamma z_a/[1 - \exp(-2\Gamma z_a)]$ for $0 < z < z_a$, and the DM map strength is given by $s = [\Delta_1\theta - \Delta_2(1-\theta)]/4$.

A pulse in a QL system is well approximated by the linear solution of Eq. (1) for large s.[8] From a Gaussian input pulse in the Fourier domain, $\hat{U}(0,\omega) = \alpha\exp(-\beta\omega^2/2)$, the linear solution is given by

$$u(z,t) = \frac{\alpha}{[2\pi(\beta + i\overline{D})]^{1/2}}\exp\left[-\frac{(t-\Omega\overline{D})^2}{2(\beta + i\overline{D})}\right] \times \exp(i\Omega t - i\overline{D}\Omega^2/2)\,, \tag{2}$$

where $\overline{D} = \overline{D}(z) = \langle D\rangle(z - z_0) + C(z)$ is the integrated dispersion and $C(z) = (1/z_a)\int_0^z \Delta(z'/z_a)\mathrm{d}z'$. In the collision model we consider two pulses of form $u_\pm(z,t)$ that have equal but opposite average frequencies $\pm\Omega$; z_0 corresponds to the average collision point of the two pulses, defined as the point of collision based on the average group velocities. With the average frequency of a pulse given by $\Omega = \mathrm{Im}[\int_{-\infty}^{\infty}(\partial u/\partial t)u^*\mathrm{d}t]/W$ and the pulse energy by $W = \int_{-\infty}^{\infty}|u|^2\mathrm{d}t = \alpha^2/(2\sqrt{\beta\pi})$, we find that the nonlinear effect of cross-phase modulation on the average frequency from Eq. (1) is $\mathrm{d}\Omega_+/\mathrm{d}z = 2g(z)\int_{-\infty}^{\infty}|u_+|^2(\partial|u_-|^2/\partial t)\mathrm{d}t/W$; integrating this equation, we obtain the equation for the FS, and integrating once more, we arrive at an expression for the TS, $\delta t(L) = \overline{D}(L)\Delta\Omega_{\mathrm{res}} + \delta t_{\mathrm{res}}$, in terms of the residual shifts. Taking $\Omega = \Omega_+$ yields

$$\Delta\Omega_{\mathrm{res}} = A\Omega \int_0^L \frac{g(z)\overline{D}}{B^3(z)} \exp\left[-2\beta \frac{\overline{D}^2\Omega^2}{B^2(z)}\right]\mathrm{d}z\,, \quad (3a)$$

$$\delta t_{\mathrm{res}} = A\Omega \int_0^L \frac{g(z)\overline{D}^2}{B^3(z)} \exp\left[-2\beta \frac{\overline{D}^2\Omega^2}{B^2(z)}\right]\mathrm{d}z\,, \quad (3b)$$

where $A = 4W\beta^{3/2}/\sqrt{2\pi}$ and $B^2(z) = \beta^2 + \overline{D}^2(z)$. In a DM system the colliding pulses follow a zigzag path, so the pulses come together, overlap, and pass through one another in a repeated fashion. The collision process thus comprises numerous mini collisions, each of which produces a small residual FS and TS that additively combine to give the total shifts for the collision process. The length of the collision process, the distance over which mini collisions (MCs) occur, is $L_C = 2s/\langle D\rangle$. There are two types of MC: that which occurs near the discontinuities of $D(z)$ or $g(z)$, which we term edge MCs, and those that occur far from a discontinuity, or internal MCs.

The integrals in Eqs. (3) are of the form $\int f(z)\exp[-\lambda\phi(z)]\mathrm{d}z$, with $\lambda \gg 1$ and $\phi(z) > 0$. Hence they are amenable to asymptotic analysis, specifically, by Laplace's method. Intuitively, for large λ the exponent decays rapidly away from the minimum points of $\phi(z)$ and the integrals can be evaluated as sums of Gaussian-type integrals. Carrying out the method shows that the minima of $\phi(z)$ correspond physically to the locations of the MCs. The MCs occur at the points z_c that satisfy the equation $\overline{D}(z_c) = \langle D\rangle(z_c - z_0) + C(z_c) = 0$. Furthermore, the location of the MC within each DM map period is given by

$$\tilde{z}_m^{(j)} = -\frac{\langle D\rangle z_a m + \overline{D}(0) + (\Delta_1 - \Delta_j)\theta/2}{\langle D\rangle z_a + \Delta_j} z_a\,, \quad (4)$$

where $z_c = mz_a + \tilde{z}_m^{(j)}$ and $-(\theta/2)z_a \le \tilde{z}_m^{(j)} \le (1 - \theta/2)z_a$ for $j = 1, 2$. Here m is the map period number and $j \in \{1, 2\}$ is the section of the DM map where the MC lies (there are two per period). The MCs locations change incrementally by $\delta\tilde{z}^{(j)} = |\tilde{z}_{m+1}^{(j)} - \tilde{z}_m^{(j)}| \simeq \langle D\rangle {z_a}^2/|\Delta_j|$.

Using the Laplace method to evaluate the residual TS of Eq. (3b), we can calculate the contribution to the TS for each internal MC as

$$\delta t_{\mathrm{MC}} \simeq W z_a g[\tilde{z}_m^{(j)}]/(2\Omega^2|\Delta_j|)\,. \quad (5)$$

The edge MCs also contribute to the residual TS, but their total contribution is small, as there are far fewer edge MCs; an accurate approximation of their contribution is obtained from the internal collisions alone. Thus the residual TS of Eq. (3b) is the sum of expression (5) over all z_c in the system.

However, for the residual FS of Eq. (3a) the contribution from internal MCs is small and is neglected. The main contributions come from the edge MCs. The contribution to the FS of the edge MCs near the discontinuity of $D(z)$ is given by

$$\Delta\Omega_{\mathrm{edge}}(z_c) = \mathrm{sgn}[\tilde{z}_m^{(j)} - \tilde{z}_e] \frac{W z_a}{\sqrt{2\beta\pi}\,\Omega\Delta_j} g[\tilde{z}_m^{(j)}] \times \exp\left\{-\frac{2}{\beta}\left(\frac{\Omega\Delta_j}{z_a}\right)^2 [\tilde{z}_e - \tilde{z}_m^{(j)}]^2\right\}, \quad (6)$$

where $\tilde{z}_e$ is the location of the discontinuity within the DM map period, $\tilde{z}_e = (\theta/2)z_a$ for the left-hand edge of the collision process, and $\tilde{z}_e = (1 - \theta/2)z_a$ for the right-hand edge. For systems with discontinuities in $g(z)$, nearby MCs also contribute to the FS. With erbium-doped fiber amplifiers positioned at the midpoint of the anomalous-dispersion segment, the contribution of each such MC is

$$\Delta\Omega_g(z_c) = \frac{2W\Gamma {z_a}^2}{\sqrt{2\beta\pi}\,\Omega\Delta_1} \exp\left\{-\frac{2}{\beta}\left(\frac{\Omega\Delta_j}{z_a}\right)^2 [\tilde{z}_m^{(j)}]^2\right\}. \quad (7)$$

To obtain the true FS it is necessary to include symmetric edge MCs calculated from Eqs. (4) and (6) for m outside the strict definition of the collision process. The total FS is the sum of all edge MCs within the system.

The figures below show the results for QL systems calculated by the Laplace method from expressions (5)–(7). We take $\alpha = 1$ and $\beta = 1$ to give a full width at half-maximum of 20 ps and a bit rate of 10 Gbits/s. The map strength is $s = 10$, average dispersion $\langle D\rangle = 0.25$, $\theta = 1/2$, and $z_a = 0.1$.

Figure 1 shows the total TS against the collision center, z_0, which is related to the initial pulse separation, t_0, by $z_0 = t_0/(\Omega\langle D\rangle)$. For ICs, only the part of the entire collision process (which extends from $z_0 - L_C/2$ to $z_0 + L_C/2$) that falls within the true physical extent of the system affects the resultant TS and FS. There are no MCs when $z_0 < -40$ and $z_0 > 65$, where the collision process does not fall within the limits of the system. The edge MCs occur at both ends of the collision process and are responsible for the peak at $z_0 = -40$, where the pulses finish colliding at the start of the system, and at $z_0 = 65$, where the pulses begin to collide at the end of the system. The results of direct

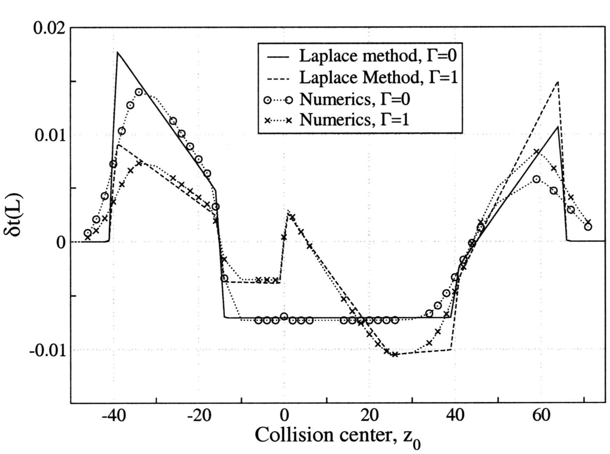

Fig. 1. Total TS versus collision center z_0 for a system with length $L = 25$; other parameters are $\langle D\rangle = 0.25$, $z_a = 0.1$, $\Omega = 5$, and $s = 10$. The TS, in physical units, is the dimensionless figure multiplied by 12 ps.

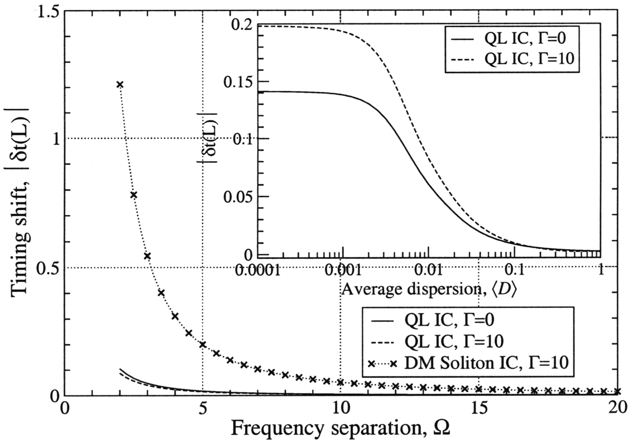

Fig. 2. Maximum TS for an incomplete collision in a QL and DM soliton system versus the dimensionless frequency separation, shown for a system with length $L = 25$; other parameters are $\langle D\rangle = 0.25$, $z_a = 0.1$, and $s = 10$. Inset, maximum TS versus average dispersion $\langle D\rangle$ at $\Omega = 10$ for a QL system.

numerical simulations of Eq. (1) with the same parameters are also shown.

Figure 2 shows the total incomplete TS versus frequency separation, Ω, for typical ICs in both lossy and lossless systems obtained by asymptotic analysis. Ω is calculated as the maximum absolute value of the TS over all initial pulse separations that result in ICs. The physical channel spacing is $f_{\text{chan}} = f_* \Omega/\pi$. The Laplace method gives good results down to $\Omega = 2$, corresponding to a channel spacing of 53 GHz. Also shown for comparison is the maximum TS for IC in a typical DM soliton system, which we obtained by applying the Laplace method to the DM soliton case. The parameters used are $s = 2.3$, $\langle D\rangle = 0.25$ and $\alpha = 2.36$, $\beta = 1$ chosen to give the same bit rate as the QL system. The QL pulse format shows a clear reduction in the maximum TS for ICs. The inset shows the maximum total TS for incomplete timing shifts versus $\langle D\rangle$; below $\langle D\rangle \simeq 0.1$ the timing shift increases before reaching a plateau.

For QL systems we can further simplify the equations for FSs and TSs by approximating the summations over the MCs as integrals. The result is accurate, as $\delta\tilde{z}^{(j)}$ is small for QL systems. For the residual TSs we approximate the summation of $g(z_c)$ as $\sum_{m,j} g[\tilde{z}_m^{(j)}]\delta\tilde{z}^{(j)} \sim \int g(z)\mathrm{d}z$. For complete collisions the integration extends from $-(\theta/2)z_a$ to $(1 - \theta/2)z_a$, and the summation of expression (5) becomes

$$\delta t_{\text{res}} \simeq W/(2\Omega^2\langle D\rangle) \tag{8}$$

for $\langle D\rangle \neq 0$. For ICs the integral extends over only a fraction of the map period $(f_a, f_b) \subset (0, 1)$, corresponding to the true range of the collision process in the system.

In a similar manner for the residual FSs we can approximate the summation of each group of MCs near a discontinuity by integrals. Near the discontinuities of $D(z)$, using Eq. (6) we get

$$\sum_m \Delta\Omega_{\text{edge}}[\tilde{z}_m^{(j)}] \sim \pm W g(\tilde{z}_e)/(\Delta_j \Omega^2 \langle D\rangle), \tag{9}$$

$\langle D\rangle \neq 0$; the sign is positive for the left-hand edge of the collision process and negative for the right-hand edge. For the MCs near the discontinuity of $g(z)$ we also approximate the sum of each group of MCs by integrals. The FS for a complete collision is given by the sum of the FSs from the left edge, the right edge, and the discontinuity point of $g(z)$. The contributions near the left–right edge discontinuity points differ in sign, so the net contribution from those points in a complete collision is negligible.

We can also obtain an elementary formula approximating the worst-case incomplete TS for a QL system. Assuming that the largest FS and TS occur together, analysis of expressions (8) and (9) for a QL system of length L gives the following convenient bound for the total TS:

$$\delta t(L) \leq \frac{W}{4s\Omega^2}\left[g_0 L + (g_0 + 2\Gamma z_a \theta)\left(L + \frac{s}{\langle D\rangle}\right)\right]. \tag{10}$$

In conclusion, we have provided an asymptotic method with which to evaluate timing shifts and frequency shifts for collisions in strongly dispersion-managed return-to-zero transmission systems. We have employed this method to analyze both quasi-linear and dispersion-managed soliton collisions. The method is valid for both incomplete and complete collisions for sufficiently large Ω. For QL systems with low average dispersion the TS reaches a maximum value. For large Ω the TS decreases as $1/\Omega^2$ (as opposed to $1/\Omega$ for classic solitons). For QL systems the results can be further simplified and a convenient formula that gives a practical worst-case TS for a QL system of any length obtained. We also found that pulses that begin colliding near the end of the system can give rise to timing shifts as great as or greater than those that arise from pulses that finish colliding near the entrance of the channel. This method can be extended to include arbitrary prechirps.

This research was partially supported by National Science Foundation grants ECS-9800152 and DMS-0101340. A. Docherty's e-mail address is dandrew@colorado.edu.

References

1. L. Mollenauer, S. Evangelides, and J. Gordon, J. Lightwave Technol. **9,** 362 (1991).
2. M. Ablowitz, G. Biondini, A. Biswas, A. Docherty, T. Hirooka, and S. Chakravarty, Opt. Lett. **27,** 318 (2002).
3. H. Sugahara and A. Maruta, J. Opt. Soc. Am. B **18,** 419 (2001).
4. Y. Kodama and A. Hasegawa, Opt. Lett. **16,** 208 (1991).
5. P. Mamyshev and L. Mollenauer, Opt. Lett. **24,** 448 (1999).
6. C. Xu, C. Xie, and L. Mollenauer, Opt. Lett. **27,** 1305 (2002).
7. R. Mu and C. Menyuk, J. Lightwave Technol. **20,** 608 (2002).
8. M. Ablowitz, T. Hirooka, and G. Biondini, Opt. Lett. **26,** 459 (2001).